Flow of Molten Glass in the Melting Chamber of a Tank Furnace

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The present paper concerns with the material and heat transfers taking into full account of the closed circulation of the convection current existing between the hot spot and the furnace wall, especially the thin layer of the stream line along the side wall demarcated by the common center of the circulations which the authors call as δ-layer. We have evaluated the velocity components both in horizontal and vertical directions as well as temperature at any point in the tank.

The results of numerical calculation have been discussed from the viewpoint of the effect of the various factors namely, X, the distance between the hot spot and the wall, Z, the depth of the tank, K_w, the overall heat transfer coefficient of the wall, Q, the pull rate of glass, and etc.

Some important conclusions are digested as follows :

Influence of X, Z, K_w. A maximum appears in the curve representing the change of h_w, the amount of heat discharge from the side wall, with increasing X, consequently there are minimums in the curve of t_m-t_w, the difference between the furnace temperature and the grand average temperature of δ-layer, and in that of the flow velocity. By the reduction of Z the flow velocity would not change in direct proportion to Z^3 as often expected, but amount to no more than a little decrease. The influence of K_w would also give rise to a small change of the flow velocity.

Influence of pull rate Q. As a large amount of additional heat should be supplied in order to bring the batch to molten glass, (t_m-t_w) and flow velocity increases with increasing Q in the compartment between the hot spot and the dog-house.

Generally it could be pointed out that the influences of any change given to the tank are not so large as one would expect at first, and this may be in debt to the correlation of many factors which are inseparably connected each other having often antagonistic trends, and acting together to reduce a disturbance.

INTRODUCTION

In continuous glass manufacturing processes batch is gradually melted in a tank furnace, whose function is to serve as a reaction chamber for the chemical reaction among the batch components, and also to serve as a container of molten glass giving sufficient time for refining and conditioning. Especially in the melting chamber all complicated chemical and physical processes occur simultaneously and successively in a space having no partition wall. On the other hand it may be regarded as a system of *heat transfer*-mechanism in

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which the top level of molten glass serves as the heat receiving surface, while the side and bottom walls act as heat discharging surfaces. Between these surfaces heat is carried across partly by conduction and radiation, and partly by the circulation of hot molten glass. Both factors determine the temperature and thus the density distribution, which in turn determine the velocity of the convection current.

It may, therefore, be easily understood that all phenomena occuring in a tank furnace are connected inseparably so that a disturbance given to one would necessarily introduce the change on others.

The purpose of the present investigation is to get a clear picture of the functions of the complicated phenomena occurring in this unaccessible space being kept at high temperature and operating under the balance of the processes which are interlocking each other and having often antagonistic trends.

Unfortunately there are but a few published data of the direct measurements, for example the temperature distribution and flow velocity in actual tank furnace, which may be used for our purpose, so that the mathematical analysis of the material and heat transfer in a tank seems to be the only possible approach to the problem. For this purpose the results of the model experiments carried out by authors during these ten years have furnished such important material that even the approximate solution of problems would never be successful without their help.

To avoid the tiresome reproduction of mathematical processes the authors confined themselves to present only the qualitative general argument paying our attention to the mile stones leading to the approximate numerical solutions.

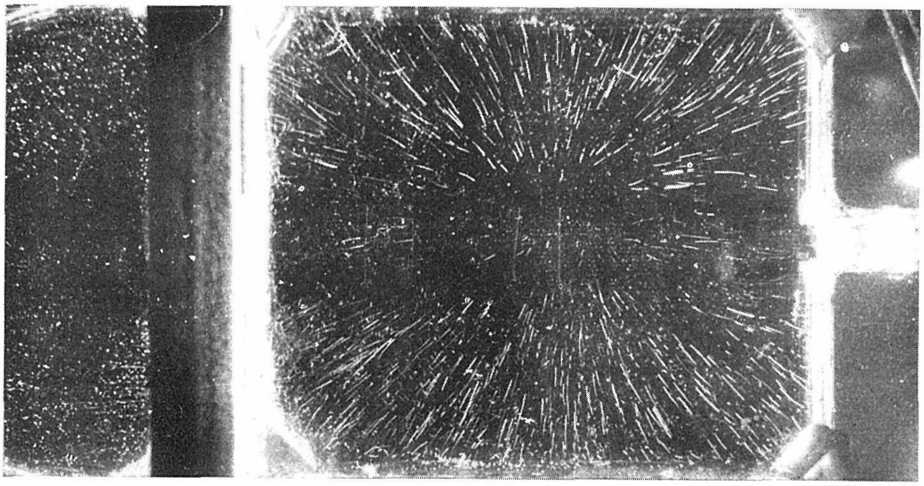
I. GENERAL FLOW PATTERN OF CONVECTION CURRENT

The keys being the most important for tackling the problem are provided by the results of the model experiments. It would therefore be convenient to introduce at first something about the general picture of the flow pattern which the authors could unveil through the series of model experiments.

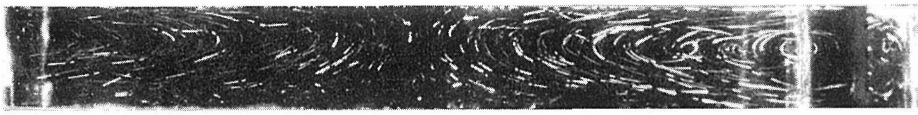
(a) **Convection current ; no pull.** The simplest case of our arguments is that of the convection current in the fluid contained in a box.

Fig. 1 shows the typical flow patterns of the liquid in the melting chamber of the 1/50 scale model of a bridge wall tank when no pull is applied, in which a) reproduces the surface current, b) the flow in the vertical surface along the longitudinal center line, c) the rapid downward stream near the wall, and d) the pattern in a plane intermediate between the wall and the hot spot.

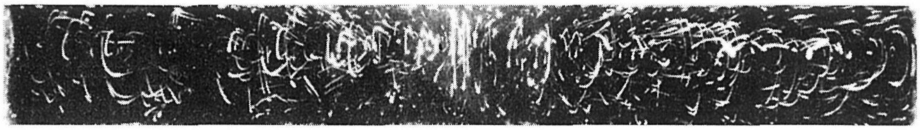
Fig. 1 a indicates clearly that the convection currents are spreading radially from the hot spot toward the walls. Fig. 1 b shows that the surface currents are the part of the circulation moving at first toward the walls, changing the direction to flow rapidly toward the bottom, and finally flowing backward toward the hot spot. In the last stage the moving liquid receives heat from above



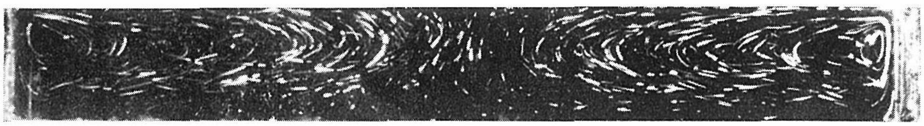
a. Surface current



b. Flow in the vertical plane along the longitudinal centerline.



c. Downward stream near the wall.



d. Flow in a plane intermediate between the hot spot and the wall.

Fig. 1. Typical flow patterns of convection current in the melting chamber of a 1/50 scale model of tank.

becoming more and more lighter through the elevation of temperature, rising gradually toward the surface until the liquid gushes out like a spring at the center.

At a glance at the picture it will be noted that the flow pattern consists of innumerable concentric closed curves whose common center is locating near the walls.

Comparing Fig. 1c and 1d it will be convinced that there exists a thin layer along the walls in which the liquid flows very rapidly. This is an important potential source for the driving force of the convection current occurring in the confined space, whereas in an infinitely extending fluid the driving force is the difference of the pressure between any two points.

Finally it should be noted that in the layer near the walls as well as at the center of the hot spot the liquid flows vertically down and upward showing that the horizontal component of the velocity is negligible. It is therefore possible to separate the currents using the vertical line at the hot spot as the center of symmetry.

(b) **Superposition of convection and pull current.** Fig. 2 reproduces the pattern obtained in an model experiment using the same model as before, but operating it at the pull rate corresponding to 6 ft^2/ton, day of the actual tank. It will be seen that the horizontal velocity is increasing by the pull, but no radical change of flow pattern occurs by this amount of pull. As already stated in the papers published before[1)2)] the effect of pull may be superposed with that of the convection current.

⟵ direction of pull

Fig. 2. Typical flow pattern in the vertical plane along the longitudinal centerline. The pull rate corresponds to 6 ft^2/ton, day in the actual tank.

As mentioned before the model experiments provide the three important keys which open the door leading to the approximate solution of the flow problems, namely :

(1) When there is no pull the convection currents may be separated into the series of closed circulations by the vertical center line of the hot spot so that it is sufficient only to consider one half of the tank. Furthermore, the flow may conveniently be treated as a simple two dimensional problem since the results may easily be extended to the actual three dimensional flow by multiplying appropriate factors.

(2) Fig. 1 shows that it is possible to set up two imaginary planes through the common center of the concentric circulations to divide the space into three

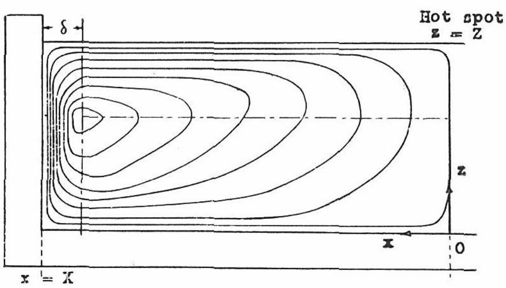


Fig. 3. Schematical rapresentation of the convection current between hot spot and wall.

as brawn schematically in Fig. 3. The thin layer between the vertical line of demarcation and the wall, which we are going to call as δ -layer, is characterized by the fact that the vertical velocity component is zero at both boundary surfaces of this layer. The horizontal line of demarcation simplifies profoundly the method of calculating the heat balance as shown later.

(3) The nature of the convection current may most easily be studied by concentrating at first our attention to the simple convection current without pull. The results obtained may be modified at the very end by adding the terms which represent the effect of pull. Unless it is stated otherwise we have limited our attention to the two dimensional flow existing in the vertical plane along the longitudinal centerline of a tank, and moreover in the space between the wall and the hot spot.

II. METHOD OF MATHEMATICAL ANALYSIS

(a) **Analysis of flow.** Convection current in a tank furnace has already been analysed by Peychés and others[3)] who have started from the solution of the simplified Navier-Stokes's equation

$$\frac{\partial p(x,z)}{\partial x} = \mu \frac{\partial^2 v_x(x,z)}{\partial x^2}, \qquad (1)$$

in which $v_x(x, z)$ is the velocity, $\frac{\partial p(x,z)}{\partial x}$ the pressure gradient both in horizontal direction x, and μ the viscosity coefficient being assumed as a constant. This equation states that the pressure gradient is the driving force which balances in a stationary state with the tangential stress due to viscosity. If the driving force in this equation may be regarded as originating from the difference of hydrostatic pressure in molten glass due to the temperature difference between two points along the same horizontal line the method of solving it becomes very simple.

This method, although simple, applies rigorously only to the liquid extending infinitely in both directions, which is not the case in actual tank.

For somewhat detailed treatment this is an oversimplification, because the effect of the rapid downward stream along the side wall should not be neglected, which however is not possible to be introduced into (1). Moreover, the flow in the enclosed circulation existing between the hot spot and the wall, whose shearing stress along the whole path of the flow should come into the question. As mentioned before the velocity component in the space between δ -layer and hot spot is very small compared with that in the δ -layer, so that the necessary additional terms for treating the problem as the flow in finite space are the vertical pressure gradient and the shearing stress in this layer.

The vertical velocity component may be formulated as

$$\frac{\partial p(x,z)}{\partial z} = -\rho(x,z)g + \mu \frac{\partial^2 v_z(x,z)}{\partial x^2} \qquad (2)$$

stating that the hydrostatic pressure gradient $\rho(x, z)g$ acts in opposite sence to the shearing stress at any point (x, z), whose algebraical sum balances with the vertical pressure gradient. In the equation $v_z(x, z)$ is the vertical velocity component of the flow, g the accelaration of gravity, and $\rho(x, z)$ the density.

The density $\rho(x, z)$ varies from point to point with the temperature of molten glass, and may be expressed as

$$\rho(x, z) = \rho_0 [1-\gamma\{t(x, z)-t_0\}] \tag{3}$$

where ρ_0 is the density at a certain reference temperature t_0, and γ is the volume expansion coefficient.

For evaluating the vertical velocity component from the equation (2) and (3) we should have to introduce the equation of continuity, which correlates the vertical and horizontal velocities,

$$\frac{\partial v_x(x, z)}{\partial x} + \frac{\partial v_z(x, z)}{\partial z} = 0, \tag{4}$$

assuming the constant density throughout the field, this assumpition introduces no serious error.

On reference to the condition at both boundary surfaces of δ-layer it is possible to formulate an equation,

$$v_w(x, z) = v_w(z)\frac{4}{\delta^3}[\delta^2\{x-(X-\delta)\}-\{x-(X-\delta)\}^3], \tag{5}$$

which gives the velocity of downward stream in this layer and corresponds to $v_z(x, z)$ in the equation (2).

In the equation δ is the thickness of the δ-layer, and X is the distance between the wall and hot spot.

Putting this into the second term of the right hand side of (2) the pressure $p(x, z)$ in δ-layer may be expressed in terms of the distance z from the bottom. In the hot spot the pressure may be approximated by the hydrostatic pressure, since the term in (2) representing the shearing stress is small. It is then possible to evaluate the pressure gradient appeared in (1).

Furthermore, if we concern ourselves only to the average velocity with respect to $x = 0 \rightarrow X$, $v_x(z)$, we may proceed to formulate an ordinary differential equation of the second order

$$\mu\frac{d^2v_x(z)}{dz^2} = \frac{1}{X}\left(\pi - \int\left\{\rho_0 g\gamma(t_s(z)-t_w(z)) + \frac{12v_w(z)f}{\delta^2}\mu_w\right\}dz\right), \tag{6}$$

where $t_s(z)$ and $t_w(z)$ are the average temperatures of the hot spot and δ-layer in the plane at the height z, μ_w the grand average of the viscosity in δ-layer, and finally f is a contant defined by $\frac{[v_x(x, z)]_{x-\delta}}{v_x(z)}$.

The simplest form of the solution of (6) is obtained by putting the integrand as a constant, and the effect of pull may be introduced from the relation

$$\int_0^z v_x(z)dz = QZ,$$

where Z is the depth of the tank, and Q the pull rate referred to an unit surface area of lateral cross section of the melting chamber, The final result is

$$v_x(z)=\frac{5\,\rho_0\,g\gamma(t_s-t_w)}{240\mu X+9f\mu_w\frac{Z^4}{\delta^3}}(15Zz^2-8z^3-6Z^2z)-\frac{3Q}{Z^2}\left(\frac{z^2}{2}-Zz\right), \tag{7}$$

in which t_s, t_w are the grand average temperature of the hot spot and δ-layer.

$v_x(z)$ thus obtained is the mean value with respect to x. The next step is to find out the velocity at any point (x, z). For this purpose we may use the results of the model experiments which suggest that $v_x\,(x, z)$ may be worked out by multiplying the first term of the right hand side of (7) with a second degree algebraical formula of x whose constants are to be determined from the results of the model experiments.

Thus the equation

$$v_x(x,z)=\frac{V}{2}(t_s-t_w)(15Zz^2-8z^3-6Z^2z)\left\{\frac{3x}{X-\delta}-\frac{3x^2}{2(X-\delta)^2}\right\}-\frac{3Q}{Z^2}\left(\frac{z^2}{2}-Zz\right) \tag{8}$$

was obtained in which

$$V=\frac{10\rho_0\,g\gamma}{240\mu X+13.5\mu_w\frac{Z^4}{\delta^3}},$$

whose denominator consists of two terms, one containing X and refers to the shearing stress due to the horizontal velocity, while the second term represents the shearing stress due to the vertical velocity in the δ-layer.

The thickness δ varies with the nature of refractory as well as of glass, and may be estimated from the experimental formula

$$\frac{\delta}{Z}=n(P_r\cdot G_r)^{-\frac{1}{4}}, \tag{9a}$$

in which n is a constant and P_r and G_r are the Prandtl and Grashof numbers with regard to the grand average temperatures, and Δt contained in Grashof number is

$$\Delta t=(t_m-t_{w0})-(t_{w0}-t_a)\frac{d}{\lambda}K_a, \tag{9b}$$

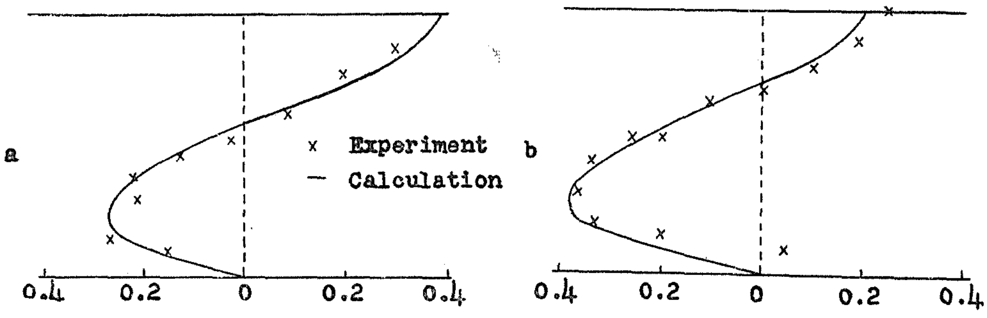


Fig. 4. Profiles of horizontal velocity components in the vertical plane along the longitudinal centerline of a model.

in which t_m is the grand average temperature, t_{w0} and t_a the average temperatures of outer surface of wall and external air, d, λ the thickness and the coefficient of thermal conductivity of wall, and K_a the heat transfer coefficient between wall and external air.

In Fig. 4 are shown $v_x(x, z)$ calculated by (8), to compare with the results of model experiments. The figures show that the agreement is satisfactory although we have made many drastic approximations.

In above discussions we have assumed that all circulations pass through δ-layer, which however is not the case if heat passes *from glass surface to super structure*, as it is often observed between hot spot and bridge wall. F.W. Preston[4] has already pointed out that in such a case the flow proceeds successively downward. As a result only a part of the flow would pass through δ-layer. This fact has the effect of reducing the velocity of flow, and consequently the vertical shearing stress in this layer. For present calculation this effect may be taken into account by assuming a thicker δ-layer.

The equation (8) gives straightforward the horizontal velocity component, from which the velocities $v_w(x, z)$ and $v_z(x, z)$ may be calculated using the equation of continuity. The results are

$$v_w(x, z) = \frac{1.5V}{\delta^4}(t_s - t_w)(5Zz^3 - 2z^4 - 3Z^2z^2)[\delta^2\{x-(X-\delta)\} - \{x-(X-\delta)\}^3], \quad (10)$$

for the flow in δ-layer, and

$$v_z(x, z) = -0.75V(t_s - t_w)(5Zz^3 - 2z^4 - 3Z^2z^2)\left\{\frac{1}{X-\delta} - \frac{x}{(X-\delta)^2}\right\}, \quad (11)$$

for the flow in the space extending from the imaginary boundary surface of the δ-layer toward the hot spot.

We are thus able to calculate the flow velocity at any point in a tank furnace if we know the temperature difference $t_s - t_w$. However, this value cannot be chosen arbitrarily but instead t_s and t_w are fixed as the natural results of the heat balance established in stationary state. In fact heat and material transfer can not be separated at our convenience. This is the reason that the authors were forced to study the heat transfer by convection current.

(b) **Hect transfer by convection current.** We are now going to discuss the problem of heat transfer in a tank furnace. For this purpose it will be convenient to list the terms which we have used in the last section, and those which we shall use in this section in a table form. Table 1 contains such terms.

In general point of view the principle is very simple, being the evaluation of heat balance which is already familiar to us. The process, however, becomes rather complicated as soon as we are forced to take account the heat carried by the molten mass of glass.

The two dimensional heat flow in stationary state is represented by

$$\rho_m c v_x(x, z)\frac{\partial t(x, z)}{\partial x} + \rho_m c v_z(x, z)\frac{\partial t(x, z)}{\partial z} - k\left(\frac{\partial^2 t(x, z)}{\partial x^2} + \frac{\partial^2 t(x, z)}{\partial z^2}\right) = 0, \quad (12)$$

in which c is the specific heat, ρ_m the average density of glass, k the radiation conductivity introduced by Kellett[5]. The equation states that the difference of the

Tabie 1. Symbols used for representing the velocities and temperatures.

$v_x(x, z)$: Horizontal velocity component as a function of x and z.

$v_x(z)=\frac{1}{X-\delta}\int_0^{X-\delta} v_x(x, z)dx$

$[v_z]=\frac{1}{D}\iint_D v_x(x, z)dxdz$: Grand average value over a domain.

$v_z(x, z)$: Vertical velocity component as a function of x and z.

$[v_z]=\frac{1}{D}\iint_D v_z(x, z)dxdz$

$v_w(x, z)$: Vertical velocity component in δ-layer as a function of x and z.

$v_w(z)=\frac{1}{\delta}\int_{X-\delta}^{X} v_w(x, z)dx$

$v_w=\frac{1}{\delta Z}\int_0^Z\int_{X-\delta}^{X} v_w(x, z)dxdz$

$t(x, z)$: Temperature as a function of x *and* z.

t_0 : Reference temperature used for expressing the density of glass.

$t_s(z)$: Temperature of glass at the hot spot.

$t_w(z)=\frac{1}{\delta}\int_{X-\delta}^{X} t(x, z)dx$

$t_s=\frac{1}{Z}\int_0^Z t_s(z)dz$

$t_m=\frac{1}{XZ}\int_0^Z\int_0^X t(x, z)dxdz$

$t_w=\frac{1}{Z}\int_0^z t_w(z)dz$

$t_m(z)=\frac{1}{X}\int_0^X t(x, z)dx$

t_{ms}, t_{mb} : Mean value of the surface and the bottom temperature $t_{ms}=t_m(Z)$, $t_{mb}=t_m(0)$.

t_{ws}, t_{wb} : Mean value of the surface and the bottom temperature in δ-layer $t_{ws}=t_w(Z)$, $t_{wb}=t_w(0)$.

t_a : Temperature of external air.

t_{w0} : Mean temperature of the outer surface of wall.

Δt : Temperature difference used in the Grashof number.

$\left(\frac{\partial t}{\partial x}\right)_m=\frac{1}{D}\iint_D \frac{\partial t(x, z)}{\partial x}dxdz$

$\left(\frac{\partial t}{\partial z}\right)_m=\frac{1}{D}\iint_D \frac{\partial t(x, z)}{\partial z}dxdz$: Grand average value over a domain.

heat quantities brought into, and taken out from a volume element by horizontal and vertical components of the flow balances with the amount of heat carried into and carried out from that element by radiation and conduction.

To begin with let us imagine a domain in the flow as shown in Fig. 5 in

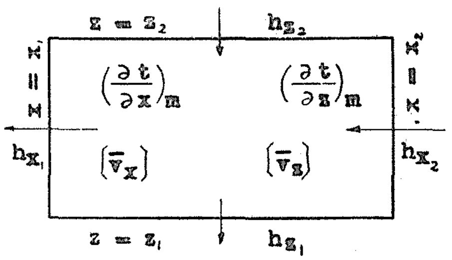


Fig. 5. An imaginary domain for the purpose of integrating the equation (12).

which the flow pattern is simple enough to calculate the mean values $[v_x]$ and $[v_z]$.

Assuming these values as constant throughout the domain and integrating (12) between boundaries using the notations inscribed in the figure the equation

$$\frac{\rho_m c}{\beta}[v_x](z_2-z_1)(x_2-x_1)\left(\frac{\partial t}{\partial x}\right)_m+\frac{\rho_m c}{\beta}[v_z](x_2-x_1)(z_2-z_1)\left(\frac{\partial t}{\partial z}\right)_m$$
$$-\{(h_{x_2}-h_{x_1})+(h_{z_2}-h_{z_1})\}=0 \qquad (13)$$

is obtained, in which $\left(\frac{\partial t}{\partial x}\right)_m$ and $\left(\frac{\partial t}{\partial z}\right)_m$ are, respectively, the average values of the temperature gradient in horizontal and vertical directions and h_{x_i}, h_{z_i}, $(i=1,2)$ are the heat quantities passing through the boundaries by radiation and conduction, and finally β is a correction factor using for compensating the errors introduced from successive simplification.

In order to apply this relation to the whole space extending between the wall and the hot spot we make use of two imaginary surfaces of demarcation to separate the tank into three compartments, namely A, B and δ, as shown in Figs. 3 and 6, which the flows are monotonous containing no flow in reverse direction. We may then work out the average values of $\left(\frac{\partial t}{\partial x}\right)_m$, $[v_x]$, and *etc.* for each compartment.

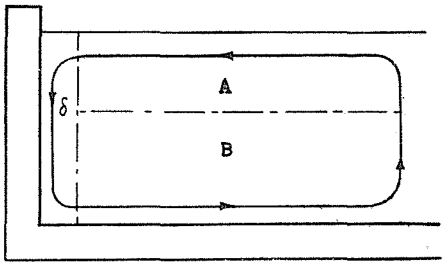


Fig. 6. Boundaries being set up for applying the equation (13) to the compartment between hot spot and wall.

However, all terms in these three compartments are inseparably correlated so that we should have to solve the three equations of heat balance simultaneously.

The first step for this purpose is to introduce the grand average temperature, t_m, which also might be presumed as the temperature to keep the molten glass in order to produce the glass of required quality under a certain pull rate. Taking into account the fact that the temperature gradient in horizontal direction is smaller than that in vertical direction we assume the linearly variation of the temperature from wall to hot spot. It is then possible to approximate the temperature difference appearing in the velocity equation (7), and *etc.*, the term (t_s-t_w) by $2(t_m-t_w)$.

The second step is to represent $\left(\frac{\partial t}{\partial x}\right)_m$'s in the compartments A and B as $\frac{2\varepsilon(t_m-t_w)}{X}$ and $\frac{2(2-\varepsilon)(t_m-t_w)}{X}$ by introducing a constant, ε, which is the ratio of the average value of the temperature difference in horizontal direction in the compartment A to (t_m-t_w).

The third step is to get the equations representing $\left(\frac{\partial t}{\partial z}\right)_m$'s. The results of the model experiments and also those of the direct measurements in actual tanks have revealed that $t_m(z)$, the average velue of $t(x, z)$ between $x=0\to X$, may be represented by

$$t_m(z)=\frac{1}{19ZK_b+48k}\left[\left\{16\frac{h_t}{X}\left(\frac{ZK_b}{k}+2\right)-16K_b(t_m-t_a)\right\}\frac{z^3}{Z^2}\right.$$
$$\left.+\left\{48K_b(t_m-t_a)-10ZK_b\frac{h_t}{Xk}\right\}z+48kt_m-10Z\frac{h_t}{X}+19ZK_bt_a\right], \qquad (14)$$

in which h_t is the heat supplied from the upper surface and K_b the overall heat transfer coefficient of the bottom. From the equation (14) it is possible to obtian the equations giving $\left(\frac{\partial t}{\partial z}\right)_m$'s.

The fourth and the last step is the evaluation of h's which may be represented as

$$h_{x_i}=\int_{z_1}^{z_2}k\left(\frac{\partial t}{\partial x}\right)_{x_i}dz, \qquad h_{z_i}=\int_{x_1}^{x_2}k\left(\frac{\partial t}{\partial z}\right)_{z_i}dx.$$

For those values in δ-layer we have introduced K_w, the overall heat transfer coefficient of the side wall.

Through the processes mentioned above the equation which gives the total heat balance of the whole space, is

$$h_t-ZK_w(t_w-t_a)-XK_b\left\{\frac{48k(t_m-t_a)-10\frac{Z}{X}h_t}{19ZK_b+48k}\right\}=0. \qquad (15)$$

The equation states that the heat received from the top surface is equal to the sum of the heats discharged through the side and bottom walls.

We have two other equations being independent each other, namely, the equation of the heat balance in A section and that in δ-layer.

The former may be written as

$$h_t-\frac{9K_bZh_t+48k\{h_t+(t_m-t_a)K_bX\}}{19ZK_b+48k}$$
$$-\frac{\rho_m c VZ^5}{\beta}\left\{\frac{\frac{h_t}{X}\left(2.43\frac{ZK_b}{k}+7.58\right)+2.72K_b(t_m-t_a)}{19ZK_b+48k}\right\}(t_m-t_w)$$
$$+\frac{\rho_m c VZ^4\varepsilon}{2\beta}(t_m-t_w)^2-\frac{kZ\varepsilon}{X}(t_m-t_w)=0, \qquad (16)$$

stating that the heat quantity supplied to the top surface is equal to the sum of the amounts, the heat passing through the boundary between A and B by radiation conductivity, the difference of heats carried into and out from this compartment by flow, and the heat carried out from this compartment to δ-layer by radiation conductivity.

Finally the heat balance in δ-layer is

$$ZK_w(t_w-t_a)-\frac{\rho_m c VZ^5}{\beta}\left\{\frac{\frac{h_t}{X}\left(1.92\frac{ZK_b}{k}+8.34\right)+6.6K_b(t_m-t_a)}{19ZK_b+48k}\right\}(t_m-t_w)$$
$$+\frac{\rho_m c VZ^4}{\beta}(1-\varepsilon)(t_m-t_w)^2-\frac{2kZ}{X}(t_m-t_w)=0, \qquad (17)$$

which gives the balance between the heat quantities, namely, discharged through the wall, the difference of those carried into and out from this layer by flow, and the heat brought into this layer by radiation conductivity.

There are five variables, *i.e.* t_m, t_w, h_t, X and ε in above equations. They were solved by giving a set of appropriate values of t_m and X. These two values, however, cannot be chosen arbitrarily, since we have divided the tank into two parts by the vertical centerline of the hot spot, and hence, we should have to repeat the same procedure for the other compartment. From the sets of values having the same t_s and from the total length of the tank L we may evaluate the t_m, t_w and h_t for both compartments.

(c) **The effects of pull.** So far we have discussed the effect of convection currents on the heat balance for the purpose of obtaining at first a clear cut picture of the problem, and we are now ready to discuss the effect of cold batch on the heat balance which is charged from dog house, melted to glass, elevated the temperature up to t_s, the temperature of hot spot, just before the molten mass arrives at this point.

To speak the truth the current in the compartment between dog house and hot spot is very complicated owing to the local currents under the mass of cold batch, and of the cold front on the bottom, both being too complicated to be dealt with precisely. The authors, therefore, have confined their attention to the *overall heat balance*.

In order to introduce the effects of pull into (15)-(17), and to consider the heat balance in some detail the amount of heat H which is necessary to convert the batch to the molten mass of t_m is divided into two: one, $(1-\xi)H$, the heat quantity supplied to the batch after once passing through the molten glass, namely, the amount which influences on the vertical temperature distribution in glass, and the other, ξH, the heat quantity which is supplied directly or indirectly through the ways without exerting any influence on the temperature distribution, which was excluded in present discussion since this was the only way to reproduce the temperature distribution obtained by actual measurements.

Furthermore, the former was again divided into two, namely: one $(1-\xi)(1-\zeta)H$ which may be included separately in the individual compartment A and B, and the other, $(1-\xi)\zeta H$, which mediates the heat balance among the three sections.

The value of the constants ξ, and ζ, may be guessed taking into account that the calculated flow velocity as well as the temperature distribution should be at least in the same order with the results of direct measurements.

Having adopted the devices described above it is possible to introduce the effects of pull by adding simply the corresponding terms to the left hand sides of (15)-(17), which are represented as E_{15}, E_{16} and E_{17}.

Thus we obtained

$$E_{15}-\rho_m cQZ(t_m-t_w)-\{q+c(t_m-t_a)\}\rho_m QZ(1-\xi)=0, \qquad (18)$$

in which q is the sum of the heat of formation of 1 kg of glass and the heat required for heating up the evolved gases to glass temperature. This equation tells that the necessary additional amount of heat for operating the tank with the pull rate of Q is given by the sum of the heat for converting the batch to the glass of t_m which is supplied after once passing through the molten glass, and the heat necessary for heating up the glass from t_m to t_s.

The equation (19) concerns with the heat balance in the compartment A, and is written as

$$E_{16}-\left[\frac{\rho_m QZ(1-\xi)(1-\zeta)}{2}\left(c\left\{\frac{\frac{h_t}{X}Z\left(\frac{3ZK_b}{k}+16\right)+(t_m-t_a)(54ZK_b+96k)}{38ZK_b+96k}\right\}+q\right)\right]$$
$$-\frac{\rho_m cQZ\varepsilon}{2}(t_m-t_w)=0, \qquad (19)$$

where the sum of the second and third terms, being enclosed in the blacket [ ], represents the heat quantity used for converting the batch to the glass of mean temperature of this compartment, and the fourth term the heat required to heat up the glass of this temperature to the temperature of the hot spot.

Finally the heat balance in δ-layer is given by

$$E_{17}-\rho_m QZ\zeta(1-\xi)\{q+c(t_m-t_a)\}=0, \qquad (20)$$

in which the second term may be regarded as a correction factor which should be introduced to E_{17} in order to complete the heat balance in A, in δ-layer, and in the whole domain.

From hot spot to throat the temperature of glass decreases gradually from t_s to t_w. Taking into account of this heat discharged from molten glass we get another three equations for the space extending from hot spot to bridge-wall.

Although we have made many bold assumptions and approximations we dare say that the above formulations including the temperatures at some important points and the flow velocities allow us to get some clear pictures of the correlations between the flows of glass and of heat.

Solving (18)-(20) we get the values of t_m, X, t_w, h_t and ε from which the mean velocity $v_x(z)$ may be worked out.

Once the average velocity was determined, it is not difficult to evaluate the velocities $v_x(x, z)$, $v_z(x, z)$ and $v_w(x, z)$ with the aid of the formulae which have been used for getting the average values. The temperature at any point may also be determined just in the same way. Hence we may conclude that the results of the above mathematical analyses open the door to the construction of the complete profile of the material and heat flow if we do not mind to take trouble of carrying out the complex and monotonous calculations.

III. RESULTS OF ANALYSIS

Having illustrated the method of mathematical analysis we are coming to the stage to discuss the matter giving examples obtained by the numerical calculations. The problem will be treated in following order :

(a) Convection current without pull.

1) Material and heat transfer in half of the trank demarcated at the hot spot.

2) Combination of two parts to form complete patterns.

(b) Effect of charging batch and pulling molten glass.

Table 2.

Figures used for the calculation $K_w=9.25\text{kcal/m}^2\text{hr}^\circ\text{C}$, $K_b=3.10\text{kcal/m}^2\text{hr}^\circ\text{C}$, $k=61.2\text{kcal/m.hr}^\circ\text{C}$ $\rho_m=2.29\times10^3\text{kg/m}^3$
$c=0.300\text{kcal/kg}^\circ\text{C}$ $t_a=30.0^\circ\text{C}$ $Z=1.00\text{m}$ $n=1.80$ $\beta=3.00$

X	$X=1\text{m}$		$X=3\text{m}$		$X=7\text{m}$		$X=3\text{m}$					
Other figures used	—		—		—		$K_w=3.31$ kcal/m²hr°C		$K_b=9.00$ kcal/m²hr°C		$Z=0.6\text{m}$	
t_m,°C	1400	1300	1400	1300	1400	1300	1400	1300	1400	1300	1400	1300
t_{ms},°C	1545	1424	1473	1368	1456	1352	1455	1351	1555	1472	1445	1336
t_{mb},°C	1314	1220	1339	1244	1347	1251	1349	1253	1266	1222	1373	1271
t_w,°C	1348	1242	1364	1260	1359	1254	1380	1279	1353	1244	1359	1256
t_{ws},°C	1322	1215	1376	1274	1376	1273	1366	1267	1307	1223	1337	1244
t_{wb},°C	1319	1226	1324	1222	1320	1216	1352	1253	1285	1230	1338	1243
t_m-t_w,°C	52	58	36	40	41	46	20	21	47	56	41	44
$t_{ms}-t_{mb}$,C	231	204	134	124	109	101	106	98	289	250	70	65
h_t, 10^3kcal/hr,m	16.17	14.90	24.52	22.67	40.86	37.82	16.73	15.5	45.59	43.41	19.77	18.34
h_t/X, 10^3kcal/hr.m²	16.17	14.90	8.18	7.56	5.84	5.41	5.58	5.17	15.19	14.46	6.59	6.12
h_w, 10^3kcal/hr.m	12.19	11.21	12.34	11.38	12.29	11.32	4.47	4.14	12.24	11.23	7.39	6.8
h_b, 10^3kcal/hr.m	3.98	3.69	12.18	11.29	28.57	26.5	12.26	11.36	33.35	32.18	12.50	11.54
h_b/X, 10^3kcal/hr.m²	3.98	3.69	4.06	3.76	3.08	3.79	4.09	3.79	11.12	10.73	4.17	3.85
δ, m	0.10	0.12	0.10	0.12	0.10	0.12	0.12	0.14	0.10	0.12	0.09	0.11
$v_x(Z)$.m/hr	4.8	4.5	3.3	3.0	3.6	2.9	3.2	2.9	4.3	4.18	3.2	2.7

h_w, h_b : Heat discharged from the wall and the bottom respectively. h_t/X, h_b/X : Heat referred to unit surface area, input from top surface and discharge from bottom. Heat quantities, h_t, h_w and h_b are referred to a part having the width of 1m.

(a) Convection Current without Pull

1) Material and heat transfer in half of the tank demarcated at the hot spot. In Table 2 are listed the figures obtained by changing X and other constants.

(i) Influences of X on h_t and temperature distribution.

As the heat supplied from the top surface is discharged from the side and bottom walls, the increase of X will give rise to the decrease of the amount of heat necessary for keeping the unit mass of glass at the same temperature, since in our two dimentional model the area of the wall remains constant while that of the top surface increases in proportion to the increase of the bottom area.

The heat input per unit area of the surface is represented by the product of the vertical temperature gradient at the surface with the radiation conductivity, and the increase of this value is reflected on the increase of the mean value of the temperature gradient in this direction. The difference of the mean temperatures at the surface t_{ms} and the bottom t_{mb} decreases with increasing X, but increases with the increasing overall heat transfer coefficients of refractories as well as the decreasing radiation conductivity of glass.

The potential source of the deviation of $t_m(z)$, the average temperature referred to x direction between the hot spot and the wall, from a straight line is the heat tranfer in horizontal direction, and the curve bends more and more with increasing heat discharge from the wall. Hence the surface temperature should not be used simply as the criterion of the temperature of the points below the surface.

(ii) Temperature distribution and the flow velocity in δ-layer.

The difference between the temperatures at the top and bottom increases with increasing amount of heat transferred by convection current. The temperature gradient in δ-layer is determined partly by this amount, and partly by the radiation conductivity of the glass. Thus, when the distance X or the overall heat transfer coefficient of the refractory is small the part of heat transferred by radiation conductivity will increase so that, in extreme case, the gradient at the surface would become even negative.

The thickness of δ-layer is governed chiefly by the radiation conductivity of glass as well as by Z and consequently the shearing stress in this layer varies with these terms.

The flow velocity in this layer is very large compared with that in other space, for example, when X=3m and t_m=1400°C its mean value is 10m/hr which is as large as the several times of the horizontal velocity component in other space and is much larger than the mean vertical velocity at the hot spot, 0.5m/hr.

(iii) The term t_m-t_w and the velocity of convection current.

Influence of X. The term t_m-t_w governs the driving force of convection current which is determined by the balance between the heat discharged from the wall and that carried by radiation and conduction. In Fig. 7 are shown the variation of the horizontal velocity component of the surface current $v_s(Z)$, the heat discharge from side wall h_w and t_m-t_w with increasing distance X, when t_m is kept constant at 1400°C.

The geometry of a tank furnace suggests that h_w would increase with increasing X if the heat input from the top surface and heat discharge from the bottom, both referred to unit area, were remain constant, since h_w is always equal to product of X and the difference of these two terms. This is indeed a factor which determines h_w.

In actual case, however, the heat discharge from bottom mentioned above does not remain constant but instead changes with X. By the balance betweem the mutual influences of the heat transfer mechanisms the amount of heat discharge from the bottom tends to increase with increasing X, which acts to reduce h_w.

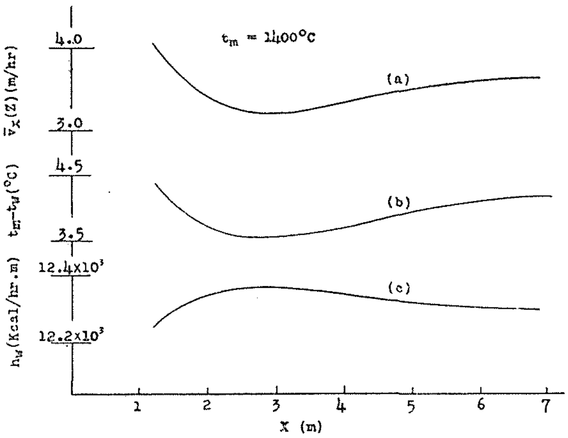


Fig. 7 Variations of the horizontal velocity component of surface current $v_x(Z)$, temperature difference t_m-t_w, and heat discharged from the wall h_w, with increasing X. Curves (a), (b), and (c) represent $v_x(Z)$, t_m-t_w, and h_w respectively.

Because of these two antagonistic factor which govern the change of h_w with the variation of X a maximum appears in curve c, and in this case, at $X=3$.

As $h_w = ZK_w(t_w - t_a) = ZK_w(t_m - t_a) - ZK_w(t_m - t_w)$,

$t_m - t_w$, and hence $v_x(Z)$ will decrease at first and then increase with further increase of X.

Influence of t_m. The figures in Table 2 suggest that $(t_m - t_w)$ decreases with increasing t_m, which could be interpreted as the result of decreasing viscocity of glass.

Influence of K_w. In the last section it was pointed out that the grand average temperature in δ-layer, t_w, determines the flow velocity in the space extending to the hot spot. If for instance, the heat transfer coefficient of the wall, K_w is reduced in order to make t_w higher the δ-layer becomes thicker, which tends to reduce the shearing stress. Consequently the flow velocity would not be reduced so much as expected from the reduction of $(t_m - t_w)$.

Influence of Z. The reduction of the depth of the tank causes the increase of shearing stress in δ-layer as well as that of the other space, since this tends to decrease the thickness of δ-layer. This process, however, meets with the changes which act in reverse direction, namely, the reduction of the path Z and increase of temperature difference $(t_m - t_w)$. Hence, in actual case the flow velocity would amount to no more than a little decrease. It is often understood that the flow velocity changes in direct proportion to Z^3. This is ture for infinitely extending liquid, but ceases to hold for actual tank, for which the effects of the side wall can not be neglected.

The figures in the Table 2 indicate clearly that the changes brought about by any given disturbance are usually smaller than our first thought. This is the result of our arguments which are based on stationary states, in which a change given rise by a disturbance would soon be counterbalanced by another change so that the second, and the new stationary state does not differ seriously from the original one. In fact a tank furnace may be regarded as an self controlling system.

2) **Combination of two parts to form complete patterns.** So far we have concerned only to the space between the wall and hot spot. Our discussion, however, would not be completed unless we take account of the correlation to the other side, since a change in one compartment should necessarily induce the disturbances in the other. Let us now discuss this problem.

Remembering that the mean temperatures, t_s, at the hot spot in two compartments, one extending from the dog house to the hot spot, and the other from the hot spot to the bridge wall should be equal, and that the sum of X's is equal to the length of the furnace, L, it is possible to correlate the changes in both compartments. Fig. 8 shows the qualitative relation between the heat input referred to unit area h_t/X and the temperature t_s of the hot spot in several combination of X and t_m. In the figure X's and t_m's are the lines of intersection of the plane t_s-h_t/X with the curved surfaces which determine the condition of stationary state together with t_s, h_t/X, and K_w. To every point on this plane there exsist the corresponding values of t_m and X given by the curves intersecting at this point. The position of the curves shifts to some degree with the change of K_w. The dotted curves in the figure represent the case when $K_w' > K_w$, where the terms, X_4, X_3, etc , and t_{m1}, t_{m2}, represent the same constant values as those of solid lines, and they increase toward the direction of arrow $(\rightarrow)$, for instance $X_4 > X_3$ and $t_{m1} > t_{m2}$. As the scale of t_s and h_t/X remains unchanged the dotted curve giving the same temperature t_m as before should be shifted upwards in conformity with the fact that the mean temperatures are lowered with increasing K_w as long as the value h_t/X be still kept constant.

To begin with let us imagine a case in which the mean temperatures in both sides of the hot spot are equal. Naturally, the distance between the walls will be halved by the hot spot having the distance X_3, and the heat input a and the mean temperature t_{m1} in both compartment are equal. If the heat input to one compartment is increased to b, the position of the hot spot will shift automatically from X_3 to X_2 nearer to the wall of the compartment as long as t_{m1} is kept unchanged. This change also brings about the change of the heat input and the mean temperature in other compartment, which may be estimated as follows :

As t_{m1} is kept constant t_s varies along the curve t_{m1}, which means that t_{s1} is elevated to t_{s1}'. A line drawn parallel to abscissa cuts the curve X_4 which is fixed by $X_4 = L - X_2$. This point determines the new heat input $c < a$ to the second compartment as well as the average temperature t_{m1}', which is higher than t_{m1}. This is the result of the change of the heat balance due to the necessary increase of the heat input rate, h_t/X, in order to reduce the distance X and set up the new stationary state.

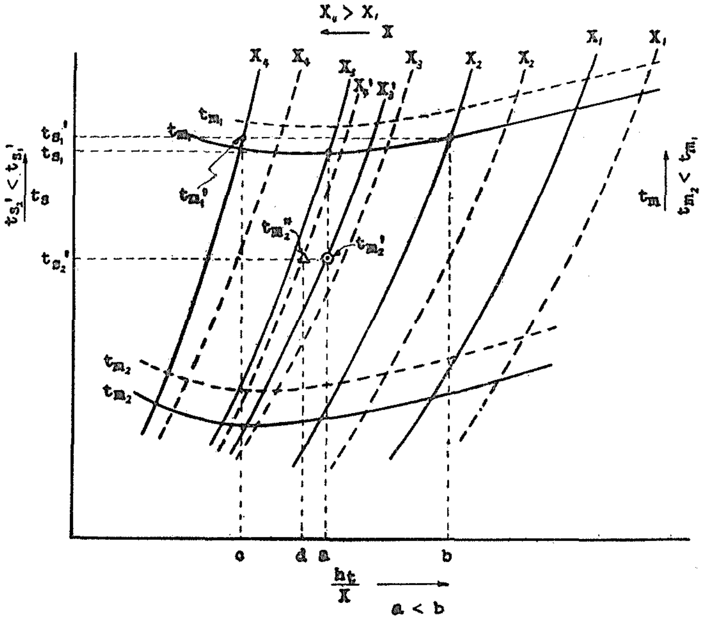


Fig. 8. Qualitative relation among the distance X, grand average temperature t_m, average temperature of the hot spot t_s, and the heat input per unit area of the top surface h_t/X. Solid and dotted curves correspond to the compartments, in which the overall heat transfer coefficient is K_w, and the larger one K_w', respectively. $2X_3 = X_2 + X_4 = X_3' + X_4'$ = total length of melting chamber L. $X_4 > X_1$, $t_{m1} > t_{m2}$, $b > a$, $t_{s1}' > t_{s2}'$.

It is interesting to know that the increase of heat input to one compartment brings out the elevation of the mean temperature of the other one to which the heat input rate is rather decreased. The reason of this apparently unreasonably trend may be illustrated as follows :

It may well be understood that a tank is divided into two parts by hot spot and the increase of the heat input rate to one compartment makes the hot spot to shift toward the wall to form new compartments, in which $(t_m - t_w)$'s could be in similar relation to curve (b) of Fig. 7 *i.e.*, X's drop in both sides of a minimum point. In such a case it is possible that $(t_m - t_w)$ in smaller compartment is larger than that in larger one, which makes t_{m1}' higher than t_{m1} since $t_m = t_s - (t_m - t_w)$, and t_s of both compartments should be the same.

Furthermore, if we increase the overall heat transfer coefficient of the wall from K_w to K_w' in one side of the hot spot the changes of its position and of the mean temperature of both compartments would occur. The relation between t_s and h_t/X is shown by the solid and dotted curves, respectively, for the compartments having the walls of K_w and K_w'. If the heat input, a, is kept unchanged as before in the compartment in which K_w remains unchanged, the mean temperature in this part would be reduced to t_{m2}' (symbol ◉) and the hot spot would come nearer to the wall changing X from X_3 to X_3'. At the same time the corresponding values in other compartment having the wall of K_w' referred to the dotted lines will be reduced, respectively, to d and t_{m2}'' (symbol △) which is lower than t_{m2}' increasing X from X_3 to X_4' and also t_s runs parallel with them.

3) **Extending to three dimensional problems.** Above calculations may be extended without difficulty to three dimensional problems.

The clue for this process is the fact that horizontal velocity is extending radially from the hot spot, and we may divide a tank into four sections introducing two imaginary vertical planes through the hot spot longitudinally and laterally.

The components in x, y, and z direction of the temperature and velocity in each compartment may be obtained if we remember the facts that t_m is equal in each compartment, and the temperatures at the boundary surface of two compartment should be equal.

The results of the calculations indicated clearly that the change is generally smaller than the two dimensional case since the third, y-component shares the influence of the disturbance.

(b) Effect of Charging Batch and Pulling Molten Glass

We are now going to touch briefly upon the influence of pull.

Needless to say that a large amount of additional heat should be supplied in order to bring the batch to the molten glass of t_s. Although we have excluded the part which does not concern in convection current the figures in Table 3 show that the temperature difference becomes larger in the compartment between the dog house and the hot spot, while that in the compartment between the hot spot and the bridge wall becomes smaller. This means that a tank furnace may be divided into two, one the heat absorbing and the other the heat discharging sections.

Table 3.

(1) Numerical values used.

$X=3.00\text{m}$, $t_m=1400°\text{C}$, $\xi=0.900$, $\zeta=0.50$, $q=100$ kcal/kg.

$$\rho_m Q = \begin{matrix} 80\times10^3 \text{ kg/6m.1m.24hr} = 555 \text{ kg/m}^2\text{hr for Case A} \\ 40\times10^3 \text{ kg/6m.1m.24hr} = 278 \text{ kg/m}^2\text{hr for Case B} \end{matrix}$$

Other constants are same as those used in Table 2.

(2) Results obtained.

In the tables the values for the compartments from dog house to hot spot and from hot spot to bridge-wall are represented by the symbols D−S and S−B, respectively.

tmp.°C		t_m	t_{ms}	t_{mb}	$t_{ms}-t_{mb}$	t_m-t_w	t_{ws}	t_{wb}
A	D−S	1400	1555	1313	242	76	1323	1289
	B−S	1400	1460	1355	105	22	1442	1332
B	D−S	1400	1510	1331	179	57	1353	1308
	S−B	1400	1466	1351	115	25	1425	1331

Velocity m/hr		Mean velocity of surface current		
		Convection	Pull	Sum $v_x(Z)$
A	D−S	6.8	−0.4	6.4
	S−B	2.0	0.4	2.4
B	D−S	5.1	−0.2	4.9
	S−B	2.3	0.2	2.1

Heat* 10^3kcal/hr.m		h_w	h_b	h_b/X	Total heat discharged from the wall and the bottom. h_w+h_b	Heat necessary to convert the batch to the glass of 1400°C. H	H(1−ξ)
A	D−S	11.96	11.93	3.98	23.89	284.1	28.41
	S−B	12.46	12.32	4.11	24.78	—	—
B	D−S	12.15	12.38	4.03	24.53	142.1	14.21
	S−B	12.45	12.56	4.10	25.01	—	—

Heat* 10^3kcal/hr.m		Heat necessary to rise the temperature of pulled glass $t_m \to t_s$.	Heat given out by pull current due to the temperature change $t_s \to t_w$.	Total heat input from the surface. h_t	** h_w+h_b+H
A	D−S	12.67	—	64.97	320.66
	S−B	—	7.34	17.44	—
B	D−S	4.75	—	43.39	171.38
	S−B	—	4.17	20.84	—

* Heat quantities are referred to a part having the width of 1m.
** Heat lost from superstructure is excluded.

The melting of batch and a part of refining are considered to occur in the first section, while the conditioning and a part of refining are shared in the second one.

Unless a considerable amount of additional heat is supplied into the first section the hot spot would move toward the bridge wall in order to receive more heat from the increased surface area, this trend is not to be recommended, since the results of the model experiments have revealed that the flow will be strongly influenced by the change of pull as soon as the hot spot comes nearer to the bridge wall than 1/3 of the width of tank. It is therefore very important to keep the hot spot at a proper position.

For melting glass the space between the dog house and the hot spot may be regarded as the factor determining, so to speak, the melting area of the tank.

Before concluding the paragraph we are going to present some schematical drawing in Fig. 9, which would help to understand the effects of several factors concerning the heat and the material transfer in a tank furnace.

The drawings show how the terms indicated with arrows vary with the increase of the term inscribed in the center circle if the terms above each figure are kept constant. The terms which increase with the increase of the term in the circle are shown in the upper half, while those which vary in reverse direction are shown in the lower half of the figure. Furthermore, the inclination of the arrows indicate qualitatively the degree of the change, so that a nearly vertical arrow suggests that the change is the largest, while a horizontal arrow suggests that practically no change occurs. For example, the drawing (a) shows that h_t and h_b will increase with the increase of X, but h_w will increase or decrease as the case may be.

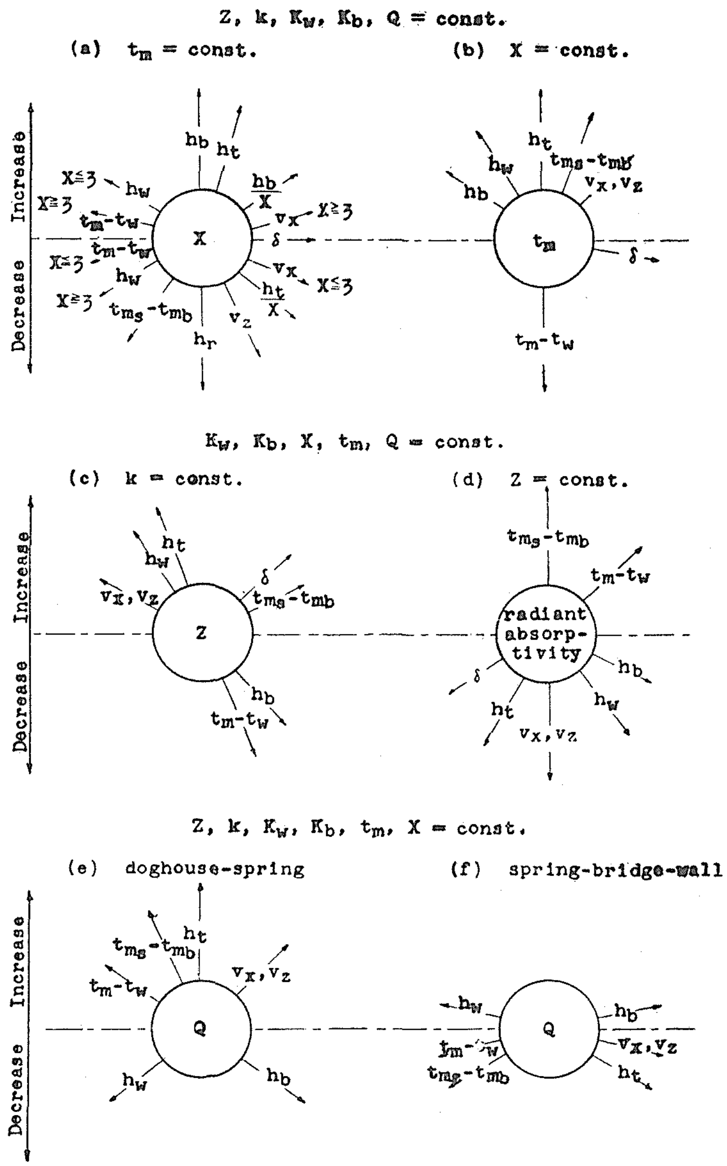


Fig. 9. Correlation among the several factors concerning the heat and the material transfer in a tank furnace.

ACKNOWLEDGEMENTS

The authors are indebted to Mr. Osamu Kuroda and to Miss Chieko Higashi who carried out the model experiments, to whom we would like to express our sincere thanks.

LIST OF NOMENCLATURES

c : Specifiic heat of glass, kcal/kg.
d : Thickness of the wall, m.
E_{15}, E_{16}, E_{17} : Left hand side of the equations (15), (16) and (17).
f : Constant in the equation (7).
g : Acceleration of gravity, m/hr^2.
H : Heat necessary to convert the batch to the molten glass of t_m, kcal/hr.
h_b : Heat discharged from the bottom of the tank, kcal/hr.
h_r : The part of the heat loss from wall which was transferred by radiation, kcal/hr
h_t : Total heat input from the surface of molten glass beetween the hot spot and the wall, kcal/hr.
h_w : Heat discharged from the wall, kcal/hr.
$h_{x1}, h_{x2}, h_{z1}, h_{z2}$: Heat transferred by radiation conductivity through the corresponding boundary surface of the domain shown in Fig. 4, kcal/hr.
k : Radiation conductivity after Kellett, kcal/hr.m. °C.
K_a : Heat transfer coefficient between the outer surface of wall and the external air, kcal/hr.m^2. °C.
K_b, K_w, K_w' : Overall heat transfer coefficients of the bottom and the wall respectively, kcal/hr.m^2. °C.
L : Length of the melting chamber, m.
n : Constant in the experimental formula (9).
$p(x,z)$: Pressure.
q : The total sum of the heat of formation of glass and the heat necessary for heating up the evolved gases, kcal/kg.
Q : Pull rate referred to an unit cross sectional area of melting chamber, m^3/m^2.hr.
$t(x,z)$: Temperature as a function of x and z, °C.
t_a : Temperature of the external air, °C.
t_m : Grand average temperature of the glass between the hot spot and the wall, °C.
$t_m(z)$: Average temperature referred to x direction between hot spot and wall, which should be represented as a function of z, °C.
t_{mb}, t_{ms} : Average temperature of bottom $t_m(0)$ and surface $t_m(Z)$, °C.
t_s : Average temperature at the hot spot, °C.
$t_s(z)$: Temperature at the hot spot as a function of z, °C.
t_w : Grand average temperature in δ-layer, °C.
t_{wo} : Average temperature of the outer surface of wall, °C.
$t_w(z)$: Average temperature as a function of z in δ-layer, °C.
t_{wb}, t_{ws} : Average temperature of bottom $t_w(0)$ and surface $t_w(Z)$ in δ-layer, °C.
t_0 : Reference temperature for expressing the density of glass, °C.

V : Coefficient with respect to the velocity of convection current introduced in the equation (8), $1/m^2.hr.\ °C$.

$v_w(x, z)$: Flow velocity in δ-layer as a function of x and z, m/hr.

$v_w(z)$: Average vertical flow velocity in δ-layer as a function of z, m/hr.

v_w : Grand average of flow velocity in δ-layer, m/hr.

$v_x(x, z)$: Horizontal component of flow velocity as a function of x and z, m/hr.

$v_x(z)$: Average of horizontal velocity componet referred to x direction as a function of z, m/hr.

$[v_x]$, $[v_z]$: Grand average of horizontal and vertical velocity components in the domain shown in Fig. 4 respectively, m/hr.

$v_z(x, z)$: Vertical component of flow velocity as a function of x and z, m/hr.

x : Horizontal coordinate, m.

X : Distance between hot spot and wall, m.

z : Vertical coordinate, m.

Z : Depth of melting chamber, m.

β : Constant as a correction factor for the simplifying procedure used in the equation (13).

γ : Coefficient of volume expansion, 1/°C.

δ : Thickness of δ-layer, m.

ε : Ratio of the average value of the temperature difference in horizontal direction in the section A to $(t_m - t_w)$.

ξ, ζ : Constants used for proportioning the heat required for melting the batch to the glass of t_m °C.

λ : Coefficient of thermal conductivity of the wall, kcal/hr.m. °C.

μ : Coefficient of viscosity of the glass at the grand average temperature t_m, kg/m.hr.

μ_w : Coefficient of viscosity of the glass at the average temperature of δ-layer t_w, kg/m.hr.

π : Constant in the equation (6).

$\rho(x, z)$: Density of glass as a function of x and z, kg/m^3.

ρ_m : Density of glass at the grand average temperature, kg/m^3.

ρ_0 : Density of glass at the reference temperature t, kg/m^3.

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